1.) In the domain of all students, we define predicates

M(x) : x is a math major

C(x) : x is a computer science major

A(x) : x is required to take CS 225 .

Express each of the following English sentences in terms of M(x), C(x), A(x), quantifiers, and logical connectives.

(a) Some computer science majors are required to take CS 225.

(b) Not all math majors are computer science majors.

(c) All students who are both computer science and math majors are not required to take CS 225.

(d) There is a student who is neither a math major and is not required to take CS 225.

2.) Let B(x), S(x), and A(x) be the predicates

B(x) : x is a good basketball player

S(x) : x is a good soccer player

A(x) : x is a good athlete

Translate each of the following quantified logic expressions (provided in the file) into English considering the domain to consist of all people .

**1.) For all people, if someone is a good athlete, then he/she is a good basketball and soccer player.**

**2.) Not everyone is a good soccer player.**

**3.) There is a person who is a good soccer and basketball player or not a good athlete.**

**4.) There is a person who is not both a good soccer player and not a good basketball player.**

3.) Negate each of the following statements:

1) Every real number is positive or negative or zero .

**There is no number that is positive or negative or zero**

2) Some people are not honest .

**All people are honest.**

3) Not all young people are good athletes.

**All young people are good athletes.**

4.) Prove or disprove that ∀x (P(x) → Q(x)) and ~∃x ~(~Q(x) → ~P(x)) are logically equivalent. ( Hint: Start with the double negation rule ~(~(∀x (P(x) → Q(x)))

**∀x (P(x) → Q(x)) ~(~(∀x (P(x) → Q(x))) by Double Negation**

**by DeMorgan’s Law for quantifiers**

**by contrapositive**

**Therefore,** ∀x (P(x) → Q(x)) and ~∃x ~(~Q(x) → ~P(x)) are logically equivalent

5.) True or false: For the set of all negative integers, ∃x (x + 1 < - x)

**TRUE. Take the case of x = -6. -6 + 1 < -(-6)  
 -5 < 6**

6.) By using direct proof method, show that if k is any odd integer and m is any even integer, then, is odd.

**Proof:**

**Suppose *k* is any odd integer and *m* is any even integer. [We must show that is odd.] By definition of odd, *k = 2a + 1* where *a* is some integer. By definition of even, *m = 2b* where *b* is some integer. By substitution and algebra,**

**Assume , which makes *t* an integer because it is the sum of the product of integers. Thus, , and by definition of odd, is odd [what we needed to show.]**

7.) Prove by contraposition that for all integers m and n, if m + n is even then m and n are both even or m and n are both odd.

If m + n is even, then ((m and n are both even) OR (m and n are both odd)

Contraposition: If ~(m and n are both odd) AND ~(m and n are both even), then m + n is odd.

**Proof (by contraposition):**

**Suppose *m* and *n* are not both odd nor both even. [We must show that *m + n* is odd.] There are two cases when *m* and *n* are not both odd nor both even. One case is when** ***m* is even and *n* is odd, and the other case is when *m* is odd and *n* is even.**

**Case 1 (*m* is odd, *n* is even):**

**Suppose m is odd and n is even. [We must show that *m + n* is odd.] By definition of odd, where *j* is some integer. By definition of even, where *k* is some integer. By substitution and algebra,**

**Assume , which makes *t* an integer because it is the sum of integers. Thus, , and by definition of odd, is odd [as we needed to show].**

**Case 2 (*m* is even, *n* is odd):**

**Suppose *m* is even, and *n* is odd. [We must show that *m + n* is odd.] By definition of even, where *a* is some integer. By definition of odd, where *b* is some integer. By substitution and algebra,**

**Assume , which makes *q* an integer because it is the sum of integers.   
Thus, and by definition of odd, is odd [as we needed to show].**

**In both cases of *m* and *n* not being both odd nor both even, we found that *m + n* is odd [as we needed to show].**